Ohio's Learning Standards | # Mathematics

Statistics and Probability-DRAFT

Ohio | Department of Education

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Introduction

PROCESS

To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education.

Then in 2019, Ohio started the Strengthening Math Pathways Initiative. Two groups were formed: the Math Pathways Advisory Council and the Math Pathways Architects. The advisory council, made of representatives from education stakeholder groups, aligned systems and structures between secondary and postsecondary mathematics. The Math Pathways Architects, made up of high school and collegiate math faculty, aligned the mathematics between the two systems. One of the outcomes was four proposed Algebra 2 equivalent courses: Quantitative Reasoning, Statistics and Probability, Data Science Foundations, and Discrete Math/Computer Science. A workgroup was formed for each of these courses. This document is the result of the Statistics and Probability Workgroup with oversight from the Math Pathways Architects.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 8 with the eight Standards for Mathematical Practice.

A Note on Rigor and Algebra 2 Equivalency

Ohio law states that students must have four units of mathematics and that one of those units should be Algebra 2/Math 3 or its equivalent. Ohio has decided to expand guidance around what it means to be *equivalent* to Algebra 2.

It has been decided that *equivalent* refers to the level of rigor and reasoning, not content. There are many branches of mathematics that are equally rigorous but have different content focuses. All equivalent courses should have the same level of rigor and reasoning that are needed to be successful in an entry-level, credit-bearing postsecondary mathematics course.

Ohio has defined rigor as the following:

> "Students use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate how, when, and why their procedure works and why it is appropriate. Students can answer the question, 'How do we know?'"

This can be illustrated in the table to the right.

Currently, four courses have been determined to be equivalent to Algebra 2: Advanced Quantitative Reasoning, Statistics and Probability, Data Science Foundations and Discrete Math/Computer Science. This document explains what should be included in an Statistics and Probability course in order to be considered Algebra 2 equivalent. Like a traditional Algebra 2 course, this course should be a year-long course in order for students to earn the credit necessary for graduation.

Rigorous courses are…	vs	Rigorous courses are not…
Defined by complexity, which is a measure of the thinking, action, or knowledge that is needed to complete the task		Characterized by difficulty, which is a measure of effort required to complete a task
Measured in depth of understanding		Measured by the amount of work
Opportunities for precision in reasoning, language, definitions, and notation that are sufficient to appropriate age/course		Based on procedure alone
Determined by students' process		Measured by assigning difficult problems
Opportunities for students to make decisions in problem solving		Defined only by the resources used
Opportunities to make connections		Taught in isolation
Supportive of the transfer of knowledge to new situations		Repetitive
Driven by students developing efficient explanations of solutions and why they work, providing opportunities for thinking and reasoning about contextual problems and situations		Focused on getting an answer
Defined by what the student does with what you give them		Defined by what you give t he student

Description of an Algebra 2 Equivalent Statistics and Probability Course

WHAT IS THE PURPOSE OF THIS COURSE?

Statistics and Probability can be used as a third or fourth mathematics credit option for students and can serve as an equivalent option for the Algebra 2 requirement. The purpose of this course is to be an introductory step in the Statistics Pathway leading to an Advanced Placement (AP) Statistics course, a College Credit Plus (CCP) Introductory Statistics course or even a high school Data Science Foundations or a CCP Introduction to Data Science course. Students who decide to pursue a major requiring Calculus will also need to take an Algebra 2 course to ensure that are on the Caclulus Pathway.

COURSE SUMMARY

In this course students collect, analyze, and interpret one- and two-variable data using statistical methods. They examine sampling techniques and experimental designs. Additionally, students expand their understanding of probability and randomness to develop an understanding of data distributions and inference for one-sample categorical and quantitative data. Each major topic will contain exploratory mathematical modeling.

GAISE II CONNECTION

Being able to reason statistically is essential in all disciplines of study and work. [The Pre-K-12 Guidelines for Assessment and Instruction in Statsitics Education II (GAISE II)] is a Framework for Statistics and Data Science Education. The statistical problem solving process is as follows:

I. Formulate Statistical Investigative Questions
II. Collect/Consider the Data
II. Analyze the Data
IV. Interpret the Results

This can be illustrated in the figure below:

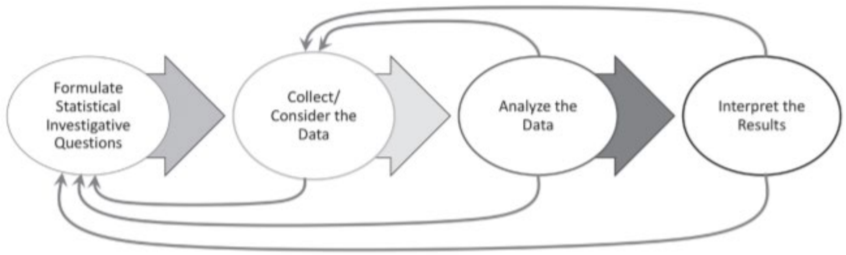


Figure : Statistical problem-solving process

The GAISE Model is a framework for all statisticsal problem solving and should not be taught in isolation. In general, the intent of this course should be to extend students from the GAISE II Level B to Level C.

HOW IS THIS COURSE DIFFERENT THAN DATA SCIENCE FOUNDATIONS?

Data Science is a cross-cutting discipline between computer science, quantitative reasoning, and statistics as applied to the real world. Data Science applies fundamental statistical principles but spends more time with large, complex data sets, including recoding to transform variables and data cleaning. Data Science includes simulation for dealing with the calculation of probabilities and analyzing the simulations using statistical tools to develop real world understanding.

There is substantial content overlap between Statistics and Data Science. However, the difference between data science and statistics is that where statistics focuses on explaining the data, data science focuses on using data to make predictions and decisions. Data science places a greater emphasis on computational thinking, and creating and interpreting visualizations of real-world processes using computational tools.

Description of an Algebra 2 Equivalent Statistics and Probability Course, continued

HOW IS THIS COURSE DIFFERENT THAN THE STATISTICS LEARNED IN PREVIOUS HIGH SCHOOL COURSES (E.G., ALGEBRA 1/GEOMETRY)?

The standards for interpreting and analyzing univariate data (S.ID.1-3) are part of the Algebra 1/Math 1 courses; although, the standard for normal distributions (S.ID.4) is not. Similarly, elementary probability standards (S.CP.1-7) are included in the Geometry/Math 2 courses. However, since statistics and probability standards are not covered in great depth and at more of an introductory level, those standards are reinforced at a greater depth and at a more advanced level in this course.

This course will now go beyond and extend those standards. The expectation for students in the previous courses is to move a student from the GAISE II Level A to Level B with statistical comprehension. In general, the intent of this course should be to extend students from the GAISE II Level B to Level C.

HOW IS THIS COURSE DIFFERENT THAN ADVANCED PLACEMENT (AP) STATISTICS (AS OF 10/4/2021)?

This course includes foundational concepts addressed in AP Statistics, making it a good on-ramp to AP Statistics for many students. Topics that commonly occur in an AP Statistics course that are **not included** in this course are the following:

- Inference for two proportions and two means
- Inference for regression
- Inference for multiple categorical variables (Chi-square)
- Verifying conditions for inference
- Type I, Type II errors and power
- Geometric distributions
- Combining random variables
- Confidence level interpretation (confidence *interval* interpretation is included)
- Z-scores with means

The following topics are touched upon in this course but will go into greater depth in AP Statistics:

- Formal inference for proportions and means
- Normal probability plots (the high school stats course would use them as a tool only if they can be easily generated with technology)
- The Central Limit Theorem
- Residual plots
- Linearizing data

HOW IS THIS COURSE DIFFERENT THAN COLLEGE CREDIT PLUS (CCP) INTRODUCTORY STATISTICS (AS OF 10/4/2021)??

The description below is based on the Ohio Transfer Module Mathematics (TMM) Standards for TMM 010-Introductory Statistics, which defines what is expected in college level statistics courses in Ohio. The learning outcomes of TMM 010-Introductory Statistics can be found on the Ohio Transfer 36: Learning Outcomes webpage.

Some topics commonly included in CCP Introductory Statistics and **not this course** may include but are not limited to the following:

- Inference for two proportions and two means
- Inference for regression
- Type I error
- Confidence level interpretation (confidence *interval* interpretation is included)
- Z-scores with means

The following topics are touched upon in this course but will go into **greater depth** in CCP Introductory Statistics:

- Formal inference for proportions and means
- The Central Limit Theorem
- Residual plots
- Linearizing data

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Standards for Mathematical Practice, continued

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Standards for Mathematical Practice, continued

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematical Content Standards for High School

PROCESS

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> *(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).*

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students. However, standards with a (+) symbol will not appear on Ohio's State Tests.

- The high school standards are listed in conceptual categories:
- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including Calculus.

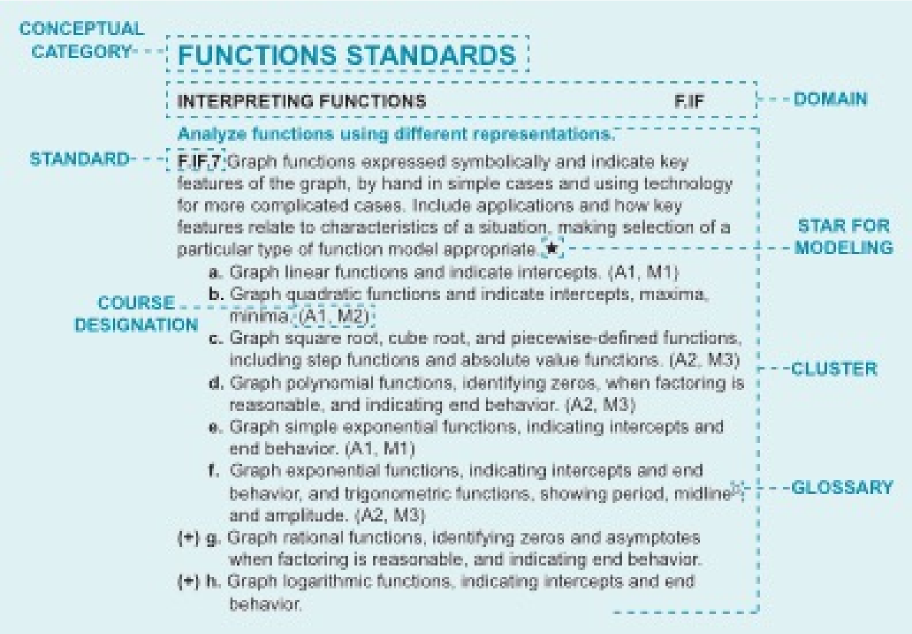
Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Proofs in high school mathematics should not be limited to geometry. Mathematically proficient high school students employ multiple proof methods, including algebraic derivations, proofs using coordinates, and proofs based on geometric transformations, including symmetries. These proofs are supported by the use of diagrams and dynamic software and are written in multiple formats including not just two-column proofs but also proofs in paragraph form, including mathematical symbols. In statistics, rather than using mathematical proofs, arguments are made based on empirical evidence within a properly designed statistical investigation.

How to Read the High School Content Standards

- **Conceptual Categories** are areas of mathematics that cross through various course boundaries.
- **Standards** define what students should understand and be able to do.
- **Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
- **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.
- [G] shows there is a definition in the glossary for this term.
- (★) indicates that modeling should be incorporated into the standard. (See the Conceptual Category of Modeling pages 60-61)
- (+) indicates that it is a standard for students who are planning on taking advanced courses. Standards with a (+) sign will not appear on Ohio's State Tests.

Some standards have course designations such as (A1, M1) or (A2, M3) listed after an **a., b.,** or **c. .**These designations help teachers know where to focus their instruction within the standard. In the example below the beginning section of the standard is the stem. The stem shows what the teacher should be doing for all courses. (Notice in the example below that modeling (★) should also be incorporated.) Looking at the course designations, an Algebra 1 teacher should be focusing his or her instruction on **a.** which focuses on linear functions; **b.** which focuses on quadratic functions; and **e.** which focuses on simple exponential functions. An Algebra 1 teacher can ignore **c., d.,** and **f,** as the focuses of these types of functions will come in later courses. However, a teacher may choose to touch on these types of functions to extend a topic if he or she wishes.



FUNCTIONS STANDARDS

INTERPRETING FUNCTIONS F.IF

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

a. Graph linear functions and indicate intercepts. (A1, M1)

b. Graph quadratic functions and indicate intercepts, maxima, minima. (A1, M2)

c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3)

d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3)

e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)

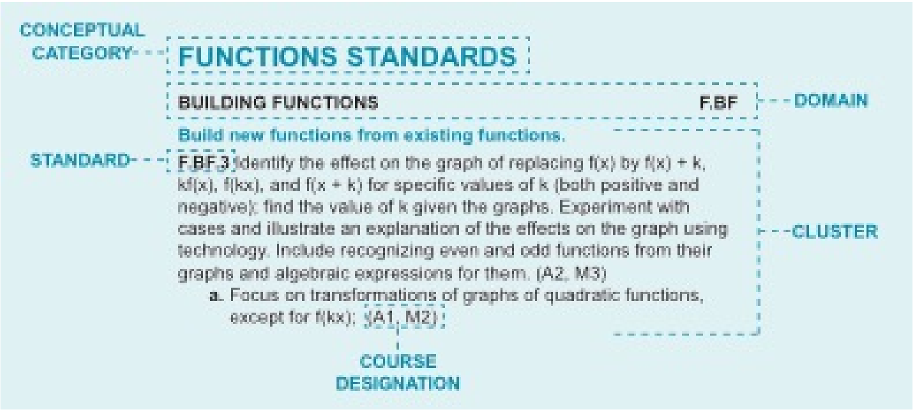
f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline[G] and amplitude. (A2, M3)

(+) g. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.

(+) h. Graph logarithmic functions, indicating intercepts and end behavior.

How to Read the High School Content Standards, continued

Notice that in the standard below, the stem has a course designation. This shows that the full extent of the stem is intended for an Algebra 2 or Math 3 course. However, **a.** shows that Algebra 1 and Math 2 students are responsible for a modified version of the stem that focuses on transformations of quadratics functions and excludes the *f(kx)* transformation. However, again a teacher may choose to touch on different types of functions besides quadratics to extend a topic if he or she wishes.



Critical Areas of Focus

CRITICAL AREA OF FOCUS #1

Communication and Analysis

Within this critical area students draw conclusions based on quantitative information and critical thinking. They recognize, make, and evaluate underlying assumptions in estimation, modeling, and data analysis. Students then organize and present thoughts and processes using mathematical and statistical evidence. They communicate clear and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation. Students demonstrate numerical reasoning orally and in writing coherent statements and paragraphs.

In the context of real-world applications, students make and investigate mathematical conjectures. They are able to defend their conjectures and respectfully question conjectures made by their classmates. This leads to the development of mathematical arguments with measures of statistical significance and confidence as ways of expressing particular kinds of reasoning and justification. Explanations (oral and written) include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language. Additionally, conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. **This critical area of focus cross cuts all the other critical areas of focus.**

Critical Area of Focus #2

Exploring One-Variable Data

Beginning in middle school, students developed an understanding of statistical problem solving at level B of the GAISE Model, displaying numerical data, using measures of center and variability, and applying those concepts in the context of real-world applications. In this course, students move to level C of the GAISE Model through the practices of gathering, displaying, summarizing, examining, and interpreting data to discover patterns and deviations from patterns. Students develop more formal methods of evaluating one-variable data and comparing two or more distributions. Students will use technology to generate and compare plots, and perform repetitive calculations.

Critical Area of Focus #3

Exploring Two-Variable Data

Beginning in middle school, students create scatterplots to recognize linear trends in data. In this course they will explore linear and non-linear data and use regression techniques to describe and interpret the relationships between quantities. Students use graphical representations and knowledge of the context to make judgements about the appropriateness of regression models. They will use algebra skills and technology to transform and plot these functions to improve the fit. They will use r^2 and residuals to analyze the goodness of fit.

Analyzing empirical situations using regression is at the heart of this unit. When functions and geometric transformations are applied, the modeling cycle should be referenced. For example, students may identify a non-linear function to model a situation, adjust parameters to improve the model, and then compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit.

CRITICAL AREA OF FOCUS #4

Collecting Data: Sampling Techniques and Experimental Design

Building on students' prior exposure to some of these concepts at Level B, student understanding will move to GAISE Level C. Students will understand what constitutes good practice in designing a sample survey, an experiment, and an observational study. This includes the role of random selection in sample surveys and random assignment in experiments as they relate to the appropriateness of conclusions. For example, students will demonstrate an understanding of the effect of sample size on the variability of estimates and the implications of random assignment for cause-and-effect interpretations.

Further, students will understand the issues of bias and confounding variables in observational studies and their implications for interpretation. Students will also understand that in some circumstances the data collected or considered may not generalize to the desired population.

CRITICAL AREA OF FOCUS #5

Probability, Randomness, and Distributions

In this course, students will build on prior learning to deepen fluency with probability, including work with compound probabilities. Additionally, students will understand the impact of probability on statistical reasoning. For example, probability will be applied by answering the question: Could the outcomes observed be due solely to chance? Students will use technology to perform simulations in an effort to more fully understand random processes.

Students will recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. They will explore expected value by defining random variables and developing a probability distribution model. Students will use technology to calculate expected value (mean) and standard deviation and use the results to evaluate outcomes of decisions.

CRITICAL AREA OF FOCUS #6

Inference for One-Sample Categorical and Quantitative Data

A foundational understanding of statistical inference is critical for understanding the practice of statistics. In this course students develop that foundation by creating margins of error and comparing treatments through simulation. These experiences of inference will include the discussion the reasonableness of generalizations. In particular they will estimate the mean or proportion of a population, given a sample from that population.

Statistics and Probability Course Overview

STATISTICS AND PROBABILITY

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

- Summarize, represent, and interpret data[G] on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.
- *Recognize and interpret the normal distribution model. (proposed)*

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments and observational studies.

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

- Understand independence and conditional probability, and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

USING PROBABILITY TO MAKE DECISIONS

- Calculate expected values, and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving the,
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NUMBER AND QUANTITY

QUANTITIES

- Reason quantitatively and use units to solve problems.

ALGEBRA

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

- Use polynomial identities to solve problems.

CREATING EQUATIONS

- Create equations that describe numbers or relationships.

FUNCTIONS

INTERPRETING FUNCTIONS

- Interpret functions that arise in applications in terms of the context.

BUILDING FUNCTIONS

- Build a function that models a relationship between two quantities.

Statistics and Probability Course Overview, continued

FUNCTIONS, CONTINUED

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

- Construct and compare linear, quadratic, and exponential models, and solve problems.
- Interpret expressions for functions in terms of the situation they model.

TRIGONOMETRIC FUNCTIONS

- Model periodic phenomena with trigonometric functions.

High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

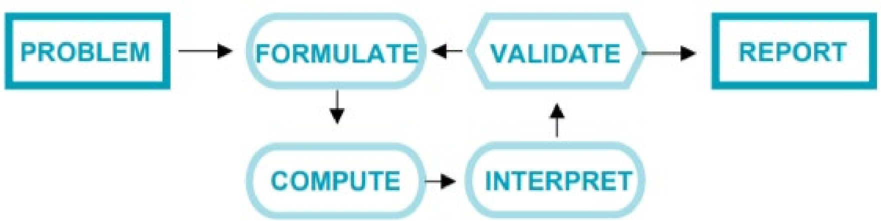
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

High School—Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

MODELING STANDARDS

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

High School—Number and Quantity

NUMBERS AND NUMBER SYSTEMS

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system— integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole number exponents leads to new and productive notation. For example, properties of whole number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)\ 3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

QUANTITIES

In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as personhours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Standards

QUANTITIES **N.Q**

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

High School—Algebra

EXPRESSIONS

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

EQUATIONS AND INEQUALITIES

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

High School—Algebra, continued

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = (\frac{(b_1+b_2)}{2})h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

CONNECTIONS WITH FUNCTIONS AND MODELING

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Standards

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS **A.APR**

Use polynomial identities to solve problems.

(+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

CREATING EQUATIONS **A.CED**

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)

High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = {}^{100}/_{v}$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph, e.g., the trace of a seismograph; by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

CONNECTIONS TO EXPRESSIONS, EQUATIONS, MODELING, AND COORDINATES.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Standards

INTERPRETING FUNCTIONS **F.IF**

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

BUILDING FUNCTIONS **F.BF**

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

- **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3)

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS **F.LE**

Construct and compare linear, quadratic, and exponential models, and solve problems.

(+) F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

TRIGONOMETRIC FUNCTIONS **F.TF**

Model periodic phenomena with trigonometric functions.

(+) F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

High School—Statistics and Probablity

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

CONNECTIONS TO FUNCTIONS AND MODELING

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Standards

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA S.ID

Summarize, represent, and interpret data[G] on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

Summarize, represent, and interpret data on two categorical and quantitative variables, continued.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

- **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3)
- **b.** Informally assess the fit of a function by discussing residuals. (A2, M3)
- **c.** Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S.ID.9 Distinguish between correlation and causation.★

Note: Standards S.ID.10-13 are included in this course to be considered for next standards revision.

Recognize and interpret the normal distribution model.

S.ID.10 *Describe and model data sets using discrete and continuous distributions. Identify distinguishing characteristics between uniform continuous, uniform discrete, binomial, and normal distributions. Calculate probabilities for discrete and continuous distributions, especially applications of the standard normal distribution.* ★

S.ID.11 *Visually compare a data distribution to the standard normal distribution. Recognize that normal distributions can be used to represent some population distributions. Understand that a normal distribution is determined by its mean and standard deviation.* ★

Statistics and Probability Standards, continued

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA, CONTINUED S.ID

Note: Standards S.ID.10-13 are included in this course to be considered for next standards revision.

Recognize and interpret the normal distribution model, continued.

S.ID.12 *Calculate and use probability from a normal distribution.* ★

- ***a.*** *Determine proportions and percentiles from a normal distribution. Understand that the 50th percentile is a measure-of-center, the median.*
- ***b.*** *Compare measures of relative position in data sets: z-scores and percentiles.*

S.ID.13 *Use the standard normal distribution to approximate binomial distributions.* ★

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS S.IC

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★

S.IC.6 Evaluate reports based on data.★

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY S.CP

Understand independence and conditional probability, and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S.CP.3 Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

Statistics and Probability Standards, continued

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY, CONTINUED S.CP

Understand independence and conditional probability, and use them to interpret data, continued.

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★

S.CP.8 Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★

Use the rules of probability to compute probabilities of compound events in a uniform probability model, continued

S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.★

USING PROBABILITY TO MAKE DECISIONS S.MD

Calculate expected values, and use them to solve problems.

S.MD.1 Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★

S.MD.2 Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★

S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★

S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★

Statistics and Probability Standards, continued

USING PROBABILITY TO MAKE DECISIONS, con't S.MD

Use probability to evaluate outcomes of decisions.

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★

- **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
- **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

S.MD.6 Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★

S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

Glossary

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1] *See also:* first quartile and third quartile.

Dot plot. *See also:* line plot.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 − 6 = 9. *See also:* first quartile, third quartile.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) *Example:* For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education. It is an updated report endorsed by the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM) to enhance the statistics standards. Like the GAISE I, it provides a framework of recommendations for developing students' foundational skills in statistical reasoning in three levels across the school years, described as levels A, B, and C. GAISE I and GAISE II can be found at https://www.amstat.org/asa/education/Guidelines-for-Assessment-and-Instruction-in-Statistics-Education-Reports.aspx.

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1. Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

2. Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

3. Adapted from Wisconsin Department of Public Instruction, *op. cit.*

4. Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Glossary, con't

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[4]

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Verify: To check the truth or correctness of a statement in specific cases.

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